Gravitational waves in an expanding Universe

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abstract

We study the tensorial modes of the two-fluid model, where one of this fluids has an equation of state $p = -\rho/3$ (variable cosmological constant, cosmic string fluid, texture) or $p = -\rho$ (cosmological constant), while the other fluid is an ordinary matter (radiation, stiff matter, incoherent matter). In the first case, it is possible to have a closed Universe whose dynamics can be that of an open Universe providing alternative solutions for the age and horizon problems. This study of the gravitational waves is extended for all values of the effective curvature $k_{eff} = k - \frac{8\pi G}{3}\rho_{0s}$, that is, positive, negative or zero, k being the curvature of the spacelike section. In the second case, we restrict ourselves to a flat spatial section. The behaviour of gravitational waves have, in each case, very particular features, that can be reflected in the anisotropy spectrum of Cosmic Microwave Background Radiation. We make also some considerations of these models as candidate to dark matter models.

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1 Introduction

Some of the main problems today in cosmology are the determination of the mass parameter Ω, the age of the Universe, and a consistent explanation of the thermal equilibrium in very early era [1, 2, 3]. The mass parameter measures the ratio between the total

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mass of the Universe and the critical mass of a spatially flat Universe. The inflationary model predicts $\Omega = 1$, i.e., a spatially flat Universe. But apparently the observational status of the mass paremeter of the Universe leads to contradictory results: The luminous mass is about $\Omega_L \sim 0.005$, while consistency with primordial nucleosynthesis suggests a baryonic mass such that $\Omega_B \sim 0.02$; however, the dynamic of motion of galaxies in a cluster, indicates a clustered mass of other $\Omega_C = 0.3$ (the indices L, B and C standing for "luminous", "baryonic" and "clustered" repectivelly); on the other hand, the position of the first doppler peak in the spectrum of anisotropy of cosmic microwave background radiation, which is generally fixed by the inverse of the square root of the total mass, indicates $\Omega_T \sim 1$, but the error bar is very large. Besides that, the deviation of the Hubble law from the linearity is consistent with a model with cosmological constant of other $\Omega_\Lambda \sim 0.7$ and $\Omega_T = 1$, leading to an accelerating Universe[4, 5]. If this is the case, we could be living now in a phase dominated by the cosmological constant.

The age problem is linked with a precise measurement of the Hubble parameter, which is yet a point of controversy, since some results point to an age for the Universe very near the measured age of globular cluster. The age of globular clusters is estimated to be of the order of $t_c \sim 15Gy$. If the Universe is now in matter dominated phase, so that $\Omega_M = 1$, the Einstein's equations imply that the scale factor behaves as $a \propto t^{2/3}$, and it results $H_0 t_0 = \frac{2}{3}$, where $H_0 = \frac{\dot{a}_0}{a_0}$ is the Hubble parameter today and t_0 is the age of the Universe. Taking $H_0 \sim 70Km/s/Mpc$, we obtain $t_0 \sim 12Gy$, that is, lesser than the age of the globular clusters. However, there is indication that better estimations of the distances can lead to smaller age for the globular clusters and a bigger age for the Universe. A more sophisticated method, taking into acount the velocity expansion, and the possible existence of a cosmological constant gives an estimation for the age of the Universe of order $t_0 = 15Gy$, while a more precise evaluation of distances of astronomical objects lead to an new estimation of the age of globular clusters to be about $t_{GC} \sim 11.5 \pm 2Gy$. But to our knowledge, these new estimations are not yet a consensus.

Finally, the thermal equilibrium of the Universe in its first moments is explained by inflating a small causally connected region to scales comparable with our observed Universe, in the so-called inflationary period [6]. Such mechanism seems necessary since otherwise, it could be difficult to understand why we observe the same temperature in regions that, at the time of emission of the photons that we receive now, was not in contact. The inflation can give an explanation for the thermal equilibrium, but there is not still at this moment an unique scenario.

All these problems may be also treated by the inclusion of non ordinary matter in the Universe. In doing so, we modify the dynamics of the Universe, consequently changing the estimation for the clustered mass and age of the Universe. In some specific cases, an alternative explanation for the isotropy of the Universe can be implemented. Very employed in these last times are the so called cold dark matter model (the effective pressure of the dark matter is zero) or hot dark matter model (the pressure is that of a radiative fluid) [7]. Observations seems to favor of the cold dark matter model. A more recent example is the so called "quintessence", a fluid component that will be present in the Universe besides the ordinary fluid. Its presence leads to an equation of state that varies from a positive (or null value) to a negative one. One realization of "quitenssence" is a scalar field Q in a slowly decreasing potential $V(Q)$. The quintessence has good consequences for the age of the Universe and leads to a spectrum of perturbations consistent with the observational data[8, 9].

In this article, we consider two main possibilities of non ordinary fluid: a stringlike fluid characterized by an equation of state $p = -\frac{\rho}{3}$, and a cosmological constant. In both cases, we consider these "extra" matters coupled to ordinary one, that is, matter characterized by an equation of state of the type $p = \alpha\rho$, with $\alpha = 0, \frac{1}{3}$ and 1. The equation of state $p = -\frac{\rho}{3}$ characterizes the limiting region from which the strong energy condition is no longer satisfied and where inflation takes place. The energy density associated with this fluid decreases as $\rho_s \propto a^{-2}$, where a is the scale factor of the Universe. Some kind of fundamental fields can be represented in some sense by such an equation of state: variable cosmological constant[10]; cosmic string[11]; texture[12]. On the other hand, the reason to consider a model with a cosmological constant term is evident from the considerations made above.

In both cases described above, there will be a period where the "extra" fluid dominates over the ordinary matter, so that the equation of state of matter evolves progressively from a positive (null) value to a negative one. We can live now in a Universe already dominated by this extra fluid. This can lead to an older Universe with respect to the Standard Cosmological Scenario, avoiding the contradictions between the age of the Universe evaluated from the Hubble law and the age of the globular clusters, for example.

In the case of a stringlike fluid, there is another nice feature. Such a fluid mimics a curvature term in the Einstein's equation: the topology of the space can be, for example, that of a closed Universe, whereas the dynamics is that of an open Universe. This can solve the horizon problem without inflation. In reference [11] the confrontation of specific models with observation leaded to some viable scenarios.

Our main interest will be concentraded in the evolution of gravitational waves in a background Universe whose matter content is one of the two fluid models described above. We will determine the solutions for an isotropic homogenous Universe. Then, we will analyse the evolution of gravitational waves in such Universe. One advantage to treat gravitational waves is that it is quite sensible to the scale factor behaviour, but the matter content does not appear directly. So, our phenomenological approach is not so decisive in the results[13]. Moreover, there is hopes that, due to the polarization of the background microwave photons, it will be possible to measure the contribution of gravitational waves to the anisotropy of CMBR, giving new tests on cosmological models.

For the stringlike fluid, we can find analytical solutions for the gravitational waves, while for the cosmological constant, we obtain analytical solutions only in the asymptotic limit. In the case of the stringlike fluid, we can define an effective cosmological constant $k_{eff} = k - \frac{8\pi\rho_{s0}}{3}$. We solve the perturbed equations for k_{eff} greater, lesser or equal to zero, and we discuss the possibility of distinguish an open Universe from a closed Universe with the dynamics of an open one. Both for the cosmological constant and stringlike fluid case, we make some considerations about them as candidate for dark matter and we analyse the implications for the deceleration (acceleration) parameter confronting it with some observational data.

The outline of this paper is a follows: in the next section we obtain the background solution for the two models; in the Section 3 we make the linear perturbative analysis in these possible Universe, and we discuss the behaviour of gravitational waves; some observational considerations are made in Section 4; our conclusions are given in Section 5.

2 Background solutions

In order to perform a more specific analysis, we will keep ourselves in the simplest case: we have two non interacting fluids, and for each of them we define an energy-momentum tensor which is conserved separately. These assumptions are consistent with those of the references quoted above. One of the energy-momentum tensor characterizes the ordinary matter (stiff matter, radiation, dust), and the other can represent a stringlike fluid or a cosmological constant. For the case of a stringlike fluid, the spatial section can be closed, open or flat. The ordinary matter has a barotropic equation of state. When the cosmological constant is treated, only a flat spatial section will be considered, since this is the scenario that seems to be favored by observations. We analyze separetely each case.

2.1 Stringlike fluid model

The equations of motion are:

$$3(\frac{\dot{a}}{a})^2 + \frac{3k}{a^2} = 8\pi G(\rho_m + \rho_s) \quad , \tag{1}$$

$$2\frac{\ddot{a}}{a} + (\frac{\dot{a}}{a})^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_s - 3\alpha\rho_m) \quad , \tag{2}$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}(1+\alpha)\rho_m = 0 \quad , \tag{3}$$

$$\dot{\rho}_s + 2\frac{\dot{a}}{a}\rho_s = 0 \quad . \tag{4}$$

In these expressions k is the curvature of the spacelike section, ρ_m is the energy density of the ordinary matter, ρ_s is the energy density of the stringlike fluid, and $p_m = \alpha\rho_m$. There is no direct interaction between the fluids. Since $\rho_s \propto a^{-2}$, we can define in equation (1) an effective curvature term that can be positive, negative or zero. The resulting equation can be written as

$$\frac{\dot{a}^2}{a^2} - \frac{k_{eff}}{a^2} = \frac{\lambda}{a^{3(1+\alpha)}} \quad , \tag{5}$$

where $k_{eff} = \frac{8\pi G}{3}\rho_{0s} - k$ and $\lambda = \frac{8\pi G}{3}\rho_{0m}$. The effective curvature term can take the values $k_{eff} = +\gamma$,$-\gamma$ or 0, where $\gamma = |\frac{8\pi G}{3}\rho_{0s} - k|$

The solutions for these equations, in terms of k_{eff} and λ and expressed in terms of the conformal time defined by $dt = ad\eta$, are straightforward and follow in the table below:

	$k_{eff} > 0$	$k_{eff} = 0$	$k_{eff} < 0$
$\alpha = -1$	$a = \sqrt{\frac{\gamma}{\lambda}} \quad \sin^{-1}(\sqrt{\gamma}\,\eta)$	$a = -(\sqrt{\lambda}\,\eta)^{-1}$	$a = -\sqrt{\frac{\gamma}{\lambda}} \quad \sinh^{-1}(\sqrt{\gamma}\,\eta)$
$\alpha = \frac{1}{3}$	$a = \sqrt{\frac{\lambda}{\gamma}}\sin(\sqrt{\gamma}\,\eta)$	$a = \sqrt{\lambda}\,\eta$	$a = \sqrt{\frac{\lambda}{\gamma}}\sinh(\sqrt{\gamma}\,\eta)$
$\alpha = 0$	$a = \frac{\lambda}{\gamma} \quad \sin^2(\frac{\sqrt{\gamma}\,\eta}{2})$	$a = \frac{\lambda\eta^2}{4}$	$a = \frac{\lambda}{\gamma} \quad \sinh^2(\frac{\sqrt{\gamma}\,\eta}{2})$
$\alpha = 1$	$a = \left(\frac{\lambda}{\gamma}\right)^{\frac{1}{4}}\sqrt{\sin(2\sqrt{\gamma}\,\eta)}$	$a = \sqrt{2\sqrt{\lambda}\,\eta}$	$a = \left(\frac{\lambda}{\gamma}\right)^{\frac{1}{4}}\sqrt{\sinh(2\sqrt{\gamma}\,\eta)}$

The effective equation of state $\alpha_{eff}(\eta) = \frac{p_T}{\rho_T}$, where ρ_T and p_T are the total density and pressure respectively, changes smoothly from the value zero (dust) or 1/3 (radiation) to $-1/3$ (stringlike fluid).

2.2 Cosmological constant model

The equations of motion, for a flat spatial section, take the form,

$$3(\frac{\dot{a}}{a})^2 = 8\pi G\rho + \Lambda \quad , \tag{6}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) \quad . \tag{7}$$

The pressure is, as before, related to the density as $p = \alpha\rho$, with $\alpha = 1$, $\frac{1}{3}$ and 0. The solutions are

1. $\alpha = 1$:
$$a = a_0 \sinh^{1/3}(\sqrt{3\Lambda}t) \quad ; \tag{8}$$

2. $\alpha = \frac{1}{3}$:
$$a = a_0 \sinh^{1/2}(2\sqrt{\frac{\Lambda}{3}}t) \quad ; \tag{9}$$

3. $\alpha = 0$:
$$a = a_0 \sinh^{2/3}(\sqrt{\frac{3\Lambda}{4}}t) \quad . \tag{10}$$

For small values of t, the ordinary matter dominates and the scale factor behaves as in the corresponding one fluid model. For large values of t, the cosmological constant dominates, and the scale factor behaves as in the de Sitter model, i.e., $a \propto e^{\sqrt{\frac{\Lambda}{3}}t}$. As in the stringlike fluid case, the effective equation of state α_{eff} evolves from 0 (dust) or 1/3 (radiation) to -1 (cosmological constant) as the Universe expands.

3 Evolution of gravitational waves

The evaluation of the perturbed quantities follows the well known approach of Lifshitz and Khalatnikov [14]. We will retain just the tensorial mode such that $h_{ij} = hQ_{ij}$, where Q_{ij} is a traceless transverse eigenfunction in the three dimensional spatial section. Perturbing the Einstein's equations, and imposing the synchronous coordinate condition, we obtain the following equation:

$$\ddot{h} - \frac{\dot{a}}{a}\dot{h} - \Big(2\frac{\ddot{a}}{a} - \frac{\bar{n}^2}{a^2}\Big)h = 0 \quad , \tag{11}$$

where $h = \frac{h_{kk}}{a^2}$, $\quad \bar{n}^2 = n^2 + 2k$,

After the conformal transformation $dt = ad\eta$ in the equation (11) we obtain the following equation:

$$h'' - 2\frac{a'}{a}h' + \Big(\bar{n}^2 - 2\frac{a''}{a} + 2\frac{a'^2}{a^2}\Big)h = 0 \quad . \tag{12}$$

where the primes mean derivatives with respect to η. We will analyze now this equation, governing the evolution of gravitational waves in an expanding Universe, for the two configurations discussed before. The integration of the equations follows standard procedure, and we present the final results only.

3.1 Stringlike fluid

The solutions of the equation (12) for different values of the k_{eff} and different phases of the evolution of the Universe are:

1. $k_{eff} > 0$

   (a) $\alpha = -1$

$$h = \sqrt{1-x^2} \quad {}_2F_1\Big(2-\sqrt{1+\tilde{n}^2}, 2+\sqrt{1+\tilde{n}^2}, \frac{5}{2}; \frac{1-x}{2}\Big) \quad , \tag{13}$$

$$\tilde{n}^2 = \frac{\bar{n}^2}{\gamma} \quad , \qquad x = \cos(\sqrt{\gamma}\,\eta) \quad ;$$

   (b) $\alpha = \frac{1}{3}$

$$h = \exp(\mp\sqrt{1+\tilde{n}^2}\,\eta) \quad \sin(\sqrt{\gamma}\,\eta) \quad , \tag{14}$$

$$\tilde{n}^2 = \frac{\bar{n}^2}{\gamma} \quad ;$$

   (c) $\alpha = 0$

$$h = \sqrt{1-x^2} \quad {}_2F_1\Big(-1-\sqrt{4+\tilde{n}^2}, -1+\sqrt{4+\tilde{n}^2}, -\frac{1}{2}; \frac{1-x}{2}\Big) \quad , \tag{15}$$

$$\tilde{n}^2 = \frac{4\bar{n}^2}{\gamma} \quad , \qquad x = \cos(\frac{\sqrt{\gamma}\,\eta}{2}) \quad ;$$

   (d) $\alpha = 1$

$$h = \sqrt{1-x^2} \quad {}_2F_1\Big(\frac{1-\sqrt{1+4\tilde{n}^2}}{2}, \frac{1+\sqrt{1+4\tilde{n}^2}}{2}, 1; \frac{1-x}{2}\Big) \quad , \tag{16}$$

$$\tilde{n}^2 = \frac{\bar{n}^2}{4\gamma} \quad , \qquad x = \cos(2\sqrt{\gamma}\,\eta) \quad ;$$

2. $k_{eff} = 0 \quad (a \propto \eta^r)$

$$h = \eta^{\frac{2r+1}{2}} \quad J_{\pm\nu}(\bar{n}\eta) \quad , \quad \nu = r + \frac{1}{2} \quad . \tag{17}$$

3. $k_{eff} < 0$

(a) $\alpha = -1$

$$\begin{aligned} h_1 &= \sqrt{x^2-1}\Big[\frac{x+1}{2}\Big]^{-2+\sqrt{1-\tilde{n}^2}} \times \\ & {}_2F_1\Big(2-\sqrt{1-\tilde{n}^2}, \frac{1}{2}-\sqrt{1-\tilde{n}^2}, 1-2\sqrt{1-\tilde{n}^2}; \frac{2}{1+x}\Big) \quad , \qquad (18) \\ & \tilde{n}^2 = \frac{\bar{n}^2}{\gamma} \quad , \qquad x = \cos(\sqrt{\gamma}\ \eta) \quad ; \end{aligned}$$

$$\begin{aligned} h_2 &= \sqrt{x^2-1}\Big[\frac{x+1}{2}\Big]^{-2-\sqrt{1-\tilde{n}^2}} \times \\ & {}_2F_1\Big(\frac{1}{2}+\sqrt{1-\tilde{n}^2}, 2+\sqrt{1-\tilde{n}^2}, 1+2\sqrt{1-\tilde{n}^2}; \frac{2}{1+x}\Big) \quad , \qquad (19) \\ & \tilde{n}^2 = \frac{\bar{n}^2}{\gamma} \quad , \qquad x = \cos(\sqrt{\gamma}\ \eta) \quad ; \end{aligned}$$

(b) $\alpha = \frac{1}{3}$

$$h = \exp(\pm\sqrt{1-\tilde{n}^2}\ \eta) \quad \sinh(\sqrt{\gamma}\ \eta) \quad , \qquad (20)$$

$$\tilde{n}^2 = \frac{\bar{n}^2}{\gamma} \quad ;$$

(c) $\alpha = 0$

$$\begin{aligned} h_1 &= \sqrt{x^2-1}\Big[\frac{x+1}{2}\Big]^{-1-\sqrt{4-\tilde{n}^2}} \times \\ & {}_2F_1\Big(-1-\sqrt{4-\tilde{n}^2}, \frac{1}{2}-\sqrt{4-\tilde{n}^2}, 1-2\sqrt{4-\tilde{n}^2}; \frac{2}{1+x}\Big) \quad , \qquad (21) \\ & \tilde{n}^2 = \frac{4\bar{n}^2}{\gamma} \quad , \qquad x = \cos(\frac{\sqrt{\gamma}\ \eta}{2}) \quad ; \end{aligned}$$

$$\begin{aligned} h_2 &= \sqrt{x^2-1}\Big[\frac{x+1}{2}\Big]^{1-\sqrt{4-\tilde{n}^2}} \times \\ & {}_2F_1\Big(\frac{1}{2}+\sqrt{4-\tilde{n}^2}, -1+\sqrt{4-\tilde{n}^2}, 1+2\sqrt{4-\tilde{n}^2}; \frac{2}{1+x}\Big) \quad , \qquad (22) \\ & \tilde{n}^2 = \frac{4\bar{n}^2}{\gamma} \quad , \qquad x = \cos(\frac{\sqrt{\gamma}\ \eta}{2}) \quad ; \end{aligned}$$

(d) $\alpha = 1$

$$h_1 = \sqrt{x^2-1}\Big[\frac{x+1}{2}\Big]^{\frac{-1+\sqrt{1-4\tilde{n}^2}}{2}} \times$$
$$ {}_2F_1\Big(\frac{1-\sqrt{1-4\tilde{n}^2}}{2}, \frac{1-\sqrt{1-4\tilde{n}^2}}{2}, 1-\sqrt{1-4\tilde{n}^2}; \frac{2}{1+x}\Big) \quad , \quad (23)$$

$$\tilde{n}^2 = \frac{\bar{n}^2}{4\gamma} \quad , \qquad x = \cos(2\sqrt{\gamma}\,\eta) \quad ;$$

$$h_2 = \sqrt{x^2-1}\Big[\frac{x+1}{2}\Big]^{\frac{-1-\sqrt{1-4\tilde{n}^2}}{2}} \times$$
$$ {}_2F_1\Big(\frac{1+\sqrt{1-4\tilde{n}^2}}{2}, \frac{1+\sqrt{1-4\tilde{n}^2}}{2}, 1+\sqrt{1-4\tilde{n}^2}; \frac{2}{1+x}\Big) \quad , \quad (24)$$

$$\tilde{n}^2 = \frac{\bar{n}^2}{4\gamma} \quad , \qquad x = \cos(2\sqrt{\gamma}\,\eta) \quad ;$$

where ${}_2F_1(a, b, c; x)$ are hypergeometric functions.

3.2 Cosmological constant

The results for the case where the cosmological constant is present are, in the asymptotical cases, those already known in the literature [15, 16]. For small values of t, the ordinary fluid dominates, and we have,

$$h \propto t^{\frac{1}{2}(r+1)} J_{\pm\nu}(n^2 t^{1-r}) \quad , \quad \nu = \frac{3r-1}{2(1-r)} \quad . \quad (25)$$

For large values of t, the cosmological constant dominates the matter content of the Universe. In this case, it is more convenient to work with the conformal time. The solution is:

$$h \propto \eta^{-1/2} J_{\pm 3/2}(n\eta) \quad . \quad (26)$$

We observe that, contrary to density perturbations, gravitational waves are produced during the deSitter phase, and in the large wavelength limit, there is a growing mode that evolves as $h \propto e^{2\sqrt{\frac{\Lambda}{3}}t}$. This contrast strongly with the gravitational waves in a matter dominated Universe, whose behaviour, in the long wavelength limit, is

$$h \propto t^{4/3} \quad . \quad (27)$$

4 Observational considerations

From the solutions described above for the stringlike fluid, it can easily be seen that, in what concerns the behaviour of gravitational waves, the difference between the sign of k_{eff} and k itself is negligible in the limit $n^2 \to \infty$, that is, for small scale perturbations. The presence of the stringlike fluid plays no significant role in this case. However, for $n^2 \to 0$, there are very important differences, and the sign of k plays an important role,

irrespective of the sign of k_{eff}. This is essential since the measure of the anisotropy of the Cosmic Microwave Background Radiation (CMBR) is very well established for small values of l, modulus the cosmic variance problem, l meaning the multipolar order in the expansion of the two points correlation function of the temperature:

$$C(\Theta) = \sum_{l=2}^{\infty} c_l P_l(\cos\Theta) \quad . \tag{28}$$

For small l, the main contribution comes from large scale perturbations, i.e., very small n^2.

In order to be more precise in our statement, we will consider a specific case in the solutions found above. For simplicity, we take the case $\alpha = \frac{1}{3}$ where the solutions for the perturbation are simpler. In all other cases, however, the reasoning is the same. Taking $k_{eff} < 0$ in the limit $n^2 \to 0$, we find,

$$h \propto e^{\pm\sqrt{1-2k}\ \eta} \sinh\sqrt{\gamma}\,\eta \quad . \tag{29}$$

Hence, we obtain the following expressions in function of the sign of k:

- $k = -1$ (open Universe):
$$h_\pm \propto e^{\pm\sqrt{3}\eta} \sinh\sqrt{\gamma}\,\eta \quad ; \tag{30}$$
- $k = 0$ (flat Universe):
$$h_\pm \propto e^{\pm\eta} \sinh\sqrt{\gamma}\,\eta \quad ; \tag{31}$$
- $k = 1$ (closed Universe)
$$h \propto \cos\eta \sinh\sqrt{\gamma}\,\eta \quad . \tag{32}$$

In the same limit, for $k_{eff} = 0$ we get the folowing expression,

- $k = 1$ (closed Universe)
$$h \propto \eta^{2r+1} \quad . \tag{33}$$

while for $k_{eff} > 0$, we get

- $k = 1$ (closed Universe)
$$h \propto e^{\pm\sqrt{3}\eta} \sin\sqrt{\gamma}\,\eta \quad . \tag{34}$$

Note that $k = 1$ admits all three possible values for k_{eff}, while $k = 0,-1$ lead to $k_{eff} < 0$.

The relevant observable quantity, the two points correlation function of the fractional fluctuation in the observed background temperature, has an expression that depends strongly on the seeds of the perturbations, and on the behaviour of perturbed quantitites, like h. If we take $k = 1$, and $k_{eff} < 0$, the behaviour of h has features completely different with respect to an open Universe. Hence, in principle, a closed Universe with a dynamics of an open one can be tested by the observation.

In the cosmological constant model, we have already seen that in the long wavelength limit, there is a very clear difference between the behaviour of gravitational waves in the matter dominated era and in cosmological constant dominated era. This must reflects in the anisotropy of CMBR provocated by a cosmological constant. We remark that, in this respect, this behaviour of gravitational wave in presence of a cosmological constant

is clearly distinct from the the behaviour of density perturbations: density perturbations generated by a cosmological constant are zero, so the determinant role is played by the ordinary fluid.

In all these cases, we must observe that the two point correlation function depends also on the geometry of the three dimensional spatial section. The eigenfunctions Q_{ij} are of course not the same if the spatial section is flat, closed or open.

The existence of a stringlike fluid or a cosmological constant may be reflected in the value of total density of the Universe, Ω_T. The observational determination of Ω remains an open problem in cosmology[2]. If the limits coming from the primordial nucleosynthesis are taking into account, the baryonic mass parameter is $\Omega_B \sim 0.02$. However, the dynamics of galaxie cluster leads to $\Omega \sim 0.3$. Moreover, the doppler peaks present in the c_l spectrum for the anisotropy of CMBR seems to be consistent with $\Omega_T \sim 1$. More recently, deviation of linearity of Hubble's law may suggest a flat Universe that is accelerating. If this result is confirmed, this is a strong evidence in favor of the existence of a cosmological constant. We remark however that the stringlike fluid may account for a fraction of dark matter of order $\Omega_s = 0.7$ only if it is a representation of a variable cosmological constant. If it represents a fluid of cosmic string, it will contribute for the clustered mass only[17]

Indeed, observations of supernova in the redshif range $0.16 < z < 0.62$ favours an accelerating Universe. What is the consequence of that for our models? The deccelerating parameter is given by $q_0 = -\frac{\ddot{a}a}{\dot{a}^2}$. We apply this expression for our two models above, for the case $\alpha = 0$, since the observations are made today.

- Cosmological constant model:

$$q_0 = 2 - \frac{3}{2}\tan^2\sqrt{\frac{3\Lambda}{4}}t \quad ; \tag{35}$$

- Stringlike fluid model ($k_{eff} < 0$):

$$q_0 = \frac{1}{2}\frac{1}{\cosh^2\frac{\sqrt{\lambda}\eta}{2}} \quad . \tag{36}$$

In the cosmological constant case, the Universe is initially decelerating, and from a time defined by

$$t_c = \sqrt{\frac{4}{3\Lambda}}\tanh^{-1}\frac{4}{9} \tag{37}$$

it begins to be accelerated. However for a stringlike fluid, the Universe is always decelerating. Hence, the confirmation of the results coming from the supernova sample may lead to discard the stringlike phenomenological model considered here, unless the observational data allow $q_0 \sim 0$ today, which is the asymptotic limit for (36).

5 Conclusions

In this article, we have discussed the evolution of the gravitational waves in the two-fluid models, consisting in the ordinary matter and the exotic matter whose equation of state is

$p = -\rho/3$ (stringlike fluid) or $p = -\rho$ (cosmological constant). For the first case, we can define an effective curvature parameter $k_{eff} = k - \lambda$, where λ is linked to the stringlike fluid density. The present study applies for all values of k_{eff}, generalizing the results of the preceding work that only treated the case $k = 1$[13]. In the second case, we have considered just a spatially flat Universe.

The solution of the linear perturbed equation, for the stringlike fluid configuration, is expressed in terms of hypergeometric functions. It comes out that the behaviour of this model, concerning gravitational waves, is strongly depending not only on the value of k_{eff} but also on the value of k. In particular for $k = 1$ the behaviour of gravitational wave is completely different if $k_{eff} = -1$, 0 or 1. For $k = 0$ and $k = -1$, we have necessarily $k_{eff} < 0$, and only in the case $k = -1$ the behaviour of gravitational wave is essentially the same as in the open Universe with no stringlike fluid. We remark that the scale factor behaviour of the background does not permit to distinguish between the sign of k and k_{eff}. But this is not the case for the gravitational waves. The most important case is when $k = 1$ and $k_{eff} < 0$, that is a closed Universe exhibiting the behaviour of an open one. Here, in the long wavelength limit, the gravitational waves behave in a complete different way with respect to an open Universe in a one fluid approach: in the last case, we have growing modes, while for the former one the amplitude of gravitational waves oscillates. In what concerns the spectrum of anisotropy of the CMBR, it is possible to distinguish all possible combinations of sign of k and k_{eff}, except $k = -1$ and $k_{eff} < 0$, due to the different expansion into harmonic functions.

For the cosmological constant model, the behaviour of gravitational waves has specific features which may permit to distinguish it from a one fluid model with ordinary matter. In particular, in the long wavelength limit the gravitational waves are strongly amplified when we enter in a phase where the cosmological constant dominates. The existence of a cosmological constant can be reflected, for example, in the position of the first doppler peak in the CMBR anisotropy spectrum, since it depends on the inverse of the square of the total mass; however, the position of the first doppler peak may indicate the existence of a dark matter, but does not reveal in principle its nature. It can be, for example, a stringlike fluid as considered here or some other exotic fluid. But, depending on the fundamental field the stringlike fluid represents, it can contribute for the clustered or unclustered mass.

Recently, however, it has been argued that analysis of a sample of supernova reveals a deviation of Hubble's law from linearity that is consistent with a cosmological model with $\Omega_T \sim 1$ and $\Omega_\Lambda = 0.7$[5]. This analysis seems to show that the Universe is an accelerating phase. If this is the case, the stringlike fluid model considered here may be disregard, since it predicts $q_0 > 0$ (decelerating Universe) unless $q_0 \sim 0$ is also allowed, which is its asymptotic limit. For the cosmological constant model, there is an initial phase for which q_0 is positive, then negative from a transition time t_c on. We note that an accelerating Universe would be a very strong indication of the existence of dark matter whose equation of state is such that $p < -\frac{\rho}{3}$ (since this equation of state implies a violation of the strong energy condition and consequently leads to an accelerated Universe), the stringlike fluid considered here being a lower limit and the cosmological constant the most natural candidate[18].

In order to have a better comparison with observations, we should calculate the spectrum of perturbations and the coefficients c_l related to the anisotropy of CMBR. This has been done for example for the case where the exotic fluid is the so called quintessence or a

variable cosmological constant[9, 19]. However, to do so, we should first evaluate density perturbations and its corresponding transfer function, and this lies outside the scope of the present work.

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